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Digital Lines & Continued Fractions

Hanna Uscka-Wehlou (Sweden)

A new recursive, continued fraction based, formula for digital lines $y = ax$ with irrational slopes $a = [0; a_1, a_2, a_3, \ldots]$

The **index jump function** corresponding to a ; **essential 1's** :

$$[0; \overset{b_1}{1}, \overset{b_2}{a_2}, \overbrace{\underline{1}, 1}^{b_3}, \overset{b_4}{a_5}, \overbrace{\underline{1}, 1}^{b_5}, \overset{b_6}{a_8}, \overset{b_7}{a_9}, \overbrace{\underline{1}, a_{11}}^{b_8}, \overset{b_9}{a_{12}}, \overbrace{\underline{1}, 1}^{b_{10}}, \overbrace{\underline{1}, a_{16}}^{b_{11}}, \overset{b_{12}}{a_{17}}, \ldots]$$

$$\downarrow$$

$$(1, 2, 3, 5, 6, 8, 9, 10, 12, 13, 15, 17, \ldots)$$

The role of **essential 1's** in the construction of runs :

- $a_{i_a(k+1)} \geq 2 \quad \Rightarrow \quad S_k$ is the main run$_k$,
- $a_{i_a(k+1)} = 1 \quad \Rightarrow \quad L_k$ is the main run$_k$.

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The pixel-wise length of digital line segments

The **denominators** of the **convergents** :

For the **convergents** $\frac{p_n}{q_n} = [0; a_1, a_2, \ldots, a_n]$ we have:
$q_0 = 1,\ q_1 = a_1,$ and, for $n \geq 2,\ \ q_n = a_n q_{n-1} + q_{n-2}$.

The **pixel-wise length** of runs on level n for the line $y = ax$:

$$|S_n| = q_{i_a(n+1)-1}, \quad |L_n| = q_{i_a(n+1)-1} + q_{i_a(n+1)-2}$$

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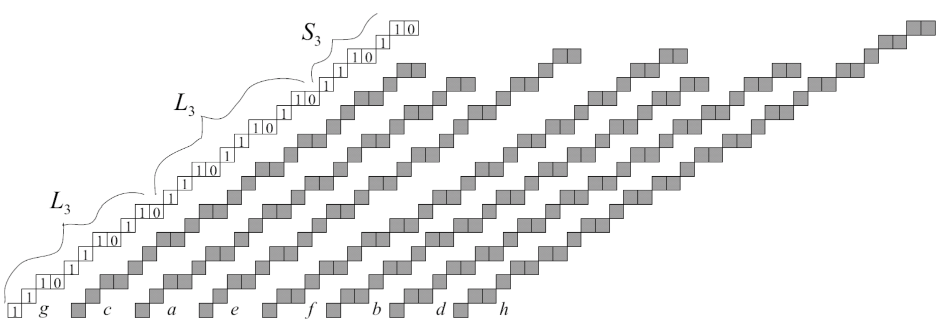
Two **equivalence relations** on the set of slopes

Quantitative (length)	Qualitative (construction)
Defined by run lengths (their cardinality)	Defined by the places of essential 1's
All lines from the same class have the same run lengths on all digitization levels.	All lines from the same class have the same construction in terms of long and short runs on all digitization levels.
An example (from paper 3) :	Theorem (from paper 3) :



$||S_1||=1,\ ||S_2||=2,\ ||S_3||=2,\ ||S_4||=3.$

$$\forall\ n \in \mathbf{N}^+\ \ [(\forall\ \ k \in [1, n-1]_{\mathbf{Z}}\ \ s_k = 2k)$$

$$\wedge\ (s_n > 2n\ \ \vee\ \ |J| = n-1)]$$

$$\Rightarrow\ \ \sup\{a;\ \ a \in [(s_j)_{j\in J}]_{\sim \text{con}}\} = \frac{F_{2n-1}}{F_{2n}},$$

where $(F_n)_{n\in\mathbf{N}^+}$ is the **Fibonacci** sequence and $(s_j)_{j\in J}$ is the sequence of the places of essential 1's.