TITLES AND ABSTRACTS

HANNA USCKA-WEHLOU

I. PUBLICATIONS:

1. ***Digital lines with irrational slopes*** (a journal paper)
Co-authors. no
Abstract. How to construct a digitization of a straight line and be able to recognize a straight line in a set of pixels are very important topics in computer graphics. The aim of the present paper is to give a mathematically exact and consistent description of digital straight lines according to Rosenfeld's definition. The digitizations of the lines with slopes $0 < a < 1$, where a is irrational, are considered. We formulate a definition of digitization runs, formulate and prove theorems containing necessary and sufficient conditions for digital straightness. The proof was successfully constructed using only methods of elementary mathematics. The developed and proved theory can be used in the research into the theory of digital lines, their symmetries, translations etc.
Number of pages. 13
Status. Published in *Theoret. Comput. Sci.* (**377** (2007) 157–169).

2. ***Continued Fractions and Digital Lines with Irrational Slopes*** (a peer-refereed conference paper)
Co-authors. no
Abstract. This paper expands on previous work on relationships between digital lines and continued fractions (CF). The main result is a parsimonious description of the construction of the digital line based only on the elements of the CF representing its slope and containing only simple integer computations. The description reflects the hierarchy of digitization runs, which raises the possibility of dividing digital lines into equivalence classes depending on the CF expansions of their slopes. Our work is confined to irrational slopes since, to our knowledge, there exists no such description for these, in contrast to rational

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slopes which have been extensively examined. The description is exact and does not use approximations by rationals. Examples of lines with irrational slopes and with very simple digitization patterns are presented. These include both slopes with periodic and non-periodic CF expansions, i.e. both quadratic surds and other irrationals.
Number of pages. 12
Status. Published (DGCI 2008, LNCS **4992**, pp. 93–104, 2008).

3. *A Run-hierarchical Description of Upper Mechanical Words with Irrational Slopes Using Continued Fractions* (a refereed conference paper)
Co-authors. no
Abstract. The main result is a run-hierarchical description (by continued fractions) of upper mechanical words with slope $a \in]0,1[\setminus \mathbf{Q}$ and intercept 0. We compare this description with two classical methods of forming of such words. In order to be able to perform the comparison, we present a quantitative analysis of our method. We use the denominators of the convergents of the continued fraction expansion of the slope to compute the length of the prefixes obtained by our method. Due to the special treatment which is given to the elements equal to 1, our method gives in some cases longer prefixes than the two other methods. Our method reflects the hierarchy of runs, by analogy to digital lines, which can give a new understanding of the construction of upper mechanical words.
Number of pages. 15
Status. Published (Proceedings of JM 2008, Mons, Belgium).

4. *Run-hierarchical structure of digital lines with irrational slopes in terms of continued fractions and the Gauss map* (a journal paper; an extended version of the conference paper 2.)
Co-authors. no
Abstract. We study relations between digital lines and continued fractions. The main result is a parsimonious description of the construction of the digital line based only on the elements of the continued fraction representing its slope and containing only simple integer computations. The description reflects the hierarchy of digitization runs, which raises the possibility of dividing digital lines into equivalence classes depending on the continued fraction expansions of their slopes. Our work is confined to irrational slopes since, to our knowledge, there exists no run-hierarchical and continued fraction based description for these, in contrast to rational slopes which have been extensively examined. The

description is exact (it does not use approximations by rationals). Examples of lines with irrational slopes and with very simple digitization patterns are presented. These include both slopes with periodic and non-periodic continued fraction expansions, i.e. both quadratic surds and other irrationals. We also derive the connection between the Gauss map and the digitization parameters introduced by the author in 2007.
Number of pages. 8

Status. Published in *Pattern Recognition* (**42** (2009) 2247–2254).

5. *Two Equivalence Relations on Digital Lines with Irrational Slopes. A Continued Fraction Approach to Upper Mechanical Words* (a journal paper)
Co-authors. no

Abstract. We examine the influence of the elements of the continued fraction (CF) expansion of $a \in \,]0,1[\, \setminus \mathbf{Q}$ on the construction of runs in the digitization of the positive half line $y = ax$ or, equivalently, on the run-hierarchical structure of the upper mechanical word with slope a and intercept 0. Special attention is given to the CF elements equal to 1. We define two complementary equivalence relations on the set of slopes, based on their CF expansions. A new description of digital lines is presented; we show how to define a straight line or upper mechanical word by two sequences of positive integers, fulfilling some extra conditions. These equivalence relations and this new description enable us to analyze the construction of digital lines and upper mechanical words.
Number of pages. 15

Status. Published in *Theoret. Comput. Sci.* (**410** (2009) 3655–3669).

6. *Sturmian words with balanced construction* (a refereed conference paper)
Co-authors. no

Abstract. In this paper we define Sturmian words with balanced construction. We formulate a fixed-point theorem for Sturmian words and analyze the set of all fixed points. The inspiration for this work came from the Kolakoski word and the general idea of self-reading sequences by Păun and Salomaa. The basis for this article is the author's earlier research on the influence of the continued fraction elements in the expansion of $a \in \,]0,1[\, \setminus \mathbf{Q}$ on the construction of runs for the upper mechanical word with slope a and intercept 0.
Number of pages. 12

Status. Published in *Proceedings of Words 2009.*

7. ***Continued Fractions, Fibonacci Numbers, and Some Classes of Irrational Numbers***
Co-authors. no

Abstract. In this paper we define an equivalence relation on the set of positive irrational numbers less than 1. The relation is defined by means of continued fractions. Equivalence classes under this relation are determined by the places of some elements equal to 1 (called *essential 1's*) in the continued fraction expansion of numbers. Analysis of suprema of all the equivalence classes leads to a solution which involves Fibonacci numbers and constitutes the main result of this paper. The problem has its origin in the author's research on the construction of digital lines and upper and lower mechanical and characteristic words according to the hierarchy of runs.
Number of pages. 14

Status. To appear in *Acta Mathematica Academiae Paedagogicae Nyíregyháziensis.*

II. <u>REPORTS</u>:

1. ***Digital lines with irrational slopes*** (an internal report)
Co-authors. no

Abstract. The same as position 1. on this list, but published as an internal report.
Number of pages. 17

Status. Scientific report (U.U.D.M. Report 2005:20).

2. ***Digital lines*** (an internal report)
Co-authors. no

Abstract. This paper is a short presentation of the first year of my research as a PhD student. I was working with Rosenfeld's digitization of straight lines. I introduced a modification of Rosenfeld's digitization and digitization parameters determining it. I showed how Euclid's algorithm for m and n describes the construction of the digital line $y = \frac{m}{n}$, where $m, n \in \mathbf{N}^+,\ m < n$ and $\gcd(m, n) = 1$. I also formulated a theorem giving an exact description of all the translations of digital lines with rational slopes.
Number of pages. 5

Status. Presentation of my research for FMB Open House.

III. <u>Ph.D. Thesis</u> (defended 25 September 2009):

Digital lines, Sturmian words, and continued fractions.
152 pp. In *Uppsala Dissertations in Mathematics* **65** (2009)
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